琉球大学理工学研究科
博士前期課程
物質地球科学専攻・物理系

入学試験問題
専門（物理）

２００７年度募集

２００６年８月１７日実施

（１）全ての解答用紙の左上に受験番号を記入のこと。
（２）大問毎に別々の解答用紙を使用すること。
（３）解答用紙は片面のみ使用のこと、縦置き横書き。
（４）用紙が足りないときは試験監督者に請求すること。
（５）問題冊子は持ち帰ること。

I 以下の各問いに答えよ．

(100 点)

問 1. 形状記憶合金で作成された，長さ r，密度一様な細い棒がある．この棒の中点を通り棒に垂直な軸のまわりに一定の角速度 ω で棒が回転している．この棒を加熱したところ，長さが n 倍になった．次の物理量は何倍になったか．

慣性モーメント．

角運動量．

回転エネルギー．

なお，棒には外力ははたらいていない．また，棒の全質量を M とおいた場合のこの回転軸のまわりの慣性モーメント I は，$\frac{1}{12}Mr^2$ となる．

問 2. ある軸のまわりに回転する慣性モーメント I の物体がある．また，軸受けの摩擦による力のモーメント N は一定である．時刻 $t=0$ のときの角速度を ω とすると，物体は何回転して止まるか．

問 3. 半径 a の滑らかな円環に，質量 m の質点が束縛されている．この円環を，一定の角速度 ω で鉛直直径のまわりに回転させる場合を考える（図. 参照）．但し，重力加速度を g とする．また，質点の位置を極座標（r, θ, φ）で表す．但し，$r=a$, $\varphi=\omega t$ である．

(1) 質点の座標を（a, θ, ωt）で表し，それを用いて質点のラグランジアン L を導出せよ．

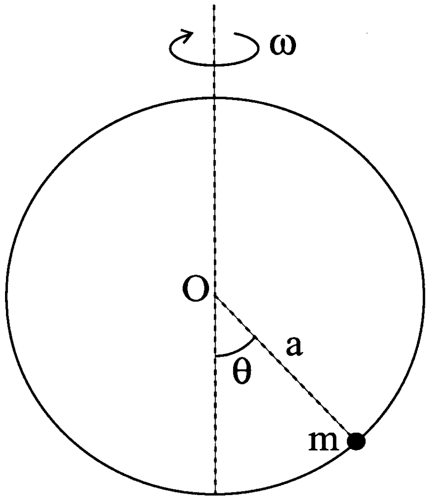
(2) 質点の θ に関するラグランジュの運動方程式は次式で表される．次式を導出せよ．

$$\ddot{\theta} = \omega^2 \sin\theta\cos\theta - \frac{g}{a}\sin\theta$$

(3) 質点が円環に対して相対的に静止する位置を求めよ．

(4) 質点が円環の最下点付近で小振動する場合の条件が $a\omega^2 < g$ であることを示せ．但し，最下点付近の微小な運動では，θ は微小であるから $\sin\theta \fallingdotseq \theta$, $\cos\theta \fallingdotseq 1$ と考えてよい．

(5) 問題 (4) の条件下において，小振動する場合の周期 T を求めよ．



II　以下の各問いに答えよ。　(100点)

問1　半径がaの無限に長い円柱があり、その円柱の内部は一定の電荷密度ρで帯電している。ただし、円柱の外部、内部の誘電率は、共にε_0とする。

(1) 円柱の中心軸からr ($r > a$) だけ離れた点 (円柱の外部) での電場の大きさを求めよ。

(2) 円柱の中心軸からr ($r < a$) だけ離れた点 (円柱の内部) での電場の大きさを求めよ。

問2　半径がRで、単位長さあたりの巻き数がnの無限に長いソレノイドに、電流Iを流したとき、ソレノイドの中心軸上の磁場の大きさはnIで与えられる。アンペールの法則により、

(1) ソレノイド内部の磁場の大きさはnIであること、

(2) ソレノイド外部の磁場の大きさは0であること、

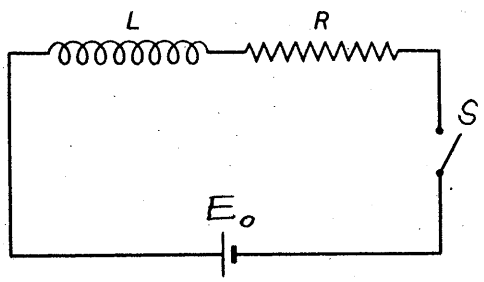
を証明せよ。

問3　図のように、起電力E_0の電池、抵抗R、自己インダクタンスLのコイル、およびスイッチSが設置されている。

(1) スイッチSを閉じた後この回路に電流Iが流れた。この回路に生じる誘導起電力V^eをLとIを用いて表せ。

(2) この回路に流れる過渡電流Iの満足する微分方程式を書け。

(3) この電流Iに対する方程式を解き、電流Iと時間tとの関係について図示せよ。



III 質量 m の粒子がかたい壁の間で一次元の運動をしている．このとき，粒子はポテンシャル

$$V(x) = \begin{cases} \infty & (x < 0) \\ 0 & (0 \leq x \leq a) \\ \infty & (a < x) \end{cases}$$

の中に束縛されていると考えてよい．次の各問いに答えよ． (100 点)

問 1 (1) 粒子の波動関数 $\psi(x,t)$ ($0 \leq x \leq a$，t は時間) の満たすシュレーディンガー方程式を書け．

(2) 波動関数が変数分離形 $\psi(x,t) = \varphi(x)f(t)$ で表されるとき，

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi(x) = E\varphi(x), \quad f(t) = e^{-iEt/\hbar}, \quad (0 \leq x \leq a)$$

が成り立つことを示せ．

(3) 上記 (2) の状況において，粒子のエネルギーを測定したとき，測定値はどうなるかについて述べよ．

問 2 問 1 の波動関数 $\varphi(x)$ に対する境界条件を $\varphi(0) = \varphi(a) = 0$ とする．

(1) エネルギー固有値が $E = 0$ のとき，$\varphi(x)$ のシュレーディンガー方程式は 0 以外の解を持たないことを示せ．

(2) エネルギー固有値が $E < 0$ のとき，$\varphi(x)$ のシュレーディンガー方程式は 0 以外の解を持たないことを示せ．

問 3 エネルギー固有値が $E > 0$ のとき，境界条件 $\varphi(0) = \varphi(a) = 0$ を用いて，問 1 の波動関数 $\varphi(x)$ のシュレーディンガー方程式を解き，

$$\varphi(x) \equiv \varphi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), \quad E \equiv E_n = \frac{\hbar^2\pi^2 n^2}{2ma^2}, \quad (n = 1, 2, 3, \cdots)$$

であることを示せ．

問 4 (1) 粒子の状態が問 3 の $\varphi_n(x)$ で表されるとき，運動量の不確定は $\Delta p = n\hbar\pi/a$ になることを示せ．ただし，$\varphi_n(x)$ は規格化されている．

(2) このとき，不確定性関係を用いて，位置の不確定の満たすべき条件を求めよ．

問 5 量子数 n が十分大きいとき，エネルギー準位の差 $\Delta E_n = E_n - E_{n-1}$ とエネルギー準位 E_n の比 $\Delta E_n / E_n$ を求め，その物理的意味を述べよ．

問 6 粒子の運動を古典的に扱い，長さ a の間で往復運動していると考える．粒子のド・ブロイ波長は，問 3 の波動関数 $\varphi_n(x)$ の波長で与えられると仮定する．

(1) 粒子の速さは $v = n\hbar\pi/(ma)$ であることを示せ．

(2) 量子数 n が十分大きいとき，問 5 のエネルギー順位の差が $\Delta E_n \simeq \hbar\omega$ を満たすことを示せ．ただし，ω は粒子の往復運動の角振動数である．

(3) 粒子の運動と固定端をもつ弦の振動との関係について述べよ．

IV 以下の各問いに答えよ。

(100点)

問1 内部エネルギーの全微分 dU は、エントロピーの微分 dS と体積の微分 dV を使って

$$dU = TdS - pdV$$

と書ける。

(1) 温度 T と圧力 p を U の偏微分で表現せよ。

(2) U を S と V で微分する時、微分の順序を交換しても結果が同じであるという条件（＝積分可能条件）から、T, S, p, V の間の関係式を作れ。

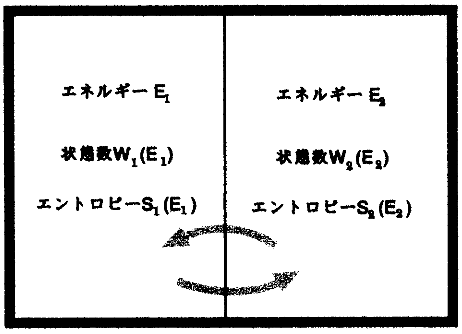
問2 エネルギー E_1、エントロピー $S_1(E_1)$、状態数 $W_1(E_1)$ の系1と、エネルギー E_2、エントロピー $S_2(E_2)$、状態数 $W_2(E_2)$ を持つ系2との間の熱平衡を考える。どちらの系も体積は変化しないとして考えよう。

(1)

二つの系を接触させて、互いにエネルギーのやりとりが可能なようにした。エネルギーは外部には漏れないとして、

$$E_{total} = E_1 + E_2$$

を一定としよう。熱力学的に考えると、二つの系が平衡に達するのは系の温度が等しくなった時である。複合系のエントロピーが最大になった時に系の温度が等しくなることを示せ。



エネルギーは自由にやりとりされる

(2) 次に平衡を統計力学的に考えたい。エントロピー S は、系の取り得る状態の数 W とボルツマン定数 k を使って $S = k \log W$ と書かれる。二つの系は独立であるとすると、複合系の状態数は積 $W_1(E_1)W_2(E_2)$ である。この複合系の状態数が最大になる時が平衡状態であるが、その条件と、(1) で考えた条件が本質的に同じことであることを示せ。

問3 次に、熱浴と平衡状態にある系の統計力学を考える。そのため、問2の系2が非常に大きな自由度を持つ系で、少々エネルギーが増減した程度では温度が変化しないような系（＝熱浴）だったとする。

(1) 系1のエネルギーが ΔE 増加した時、系2のエネルギーは ΔE 減少するが、この時系2のエントロピーと状態数はそれぞれどう変化するか。変化量もしくは何倍になるかを答えよ。

(2) 系1のエネルギーが0であった時、系2の状態数が W_0 だったとすると、系1がエネルギー E を持つとき、系2の状態数はどれだけになっていると考えられるか。

(3) 等確率の法則（等重率の法則）からすると、あらゆる微視的状態は同じ確率で実現する筈である。ところが、今考えている系で系1の取る状態を一つ指定したとすると、その状態が実現する確率は系1のエネルギーが大きいほど小さくなるという。それはなぜか、上の結果を使って説明せよ。

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別紙

Einstein's Visit in Asia

Cont. from No. 2, Vol. 15

Michiji Konuma, Guest Editor

Einstein's Impact on Japanese Culture

Kaneko Tsutomu[1]

1. EXCITEMENT ON EINSTEIN

Departing for Japan by the N. Y. K. (Nippon Yusen Kabushiki-kaisha) liner Kitanomaru which sailed from Marseille on October 8, 1922, Albert Einstein arrived in Kobe, Japan, on November 17, 1922 and stayed for forty-three days in this Far East land, sparking off an "Einstein boom" in Taisho Japan (1912-1926). Sailing from Moji on the N. Y. K. liner *Harunamaru* which departed on December 29, Einstein arrived on February 1 in the following year at Port Said, Egypt where he went ashore and headed for Palestine. Including the time spent aboard ship on the outward and homeward voyages, his visit to Japan lasted a full four months.

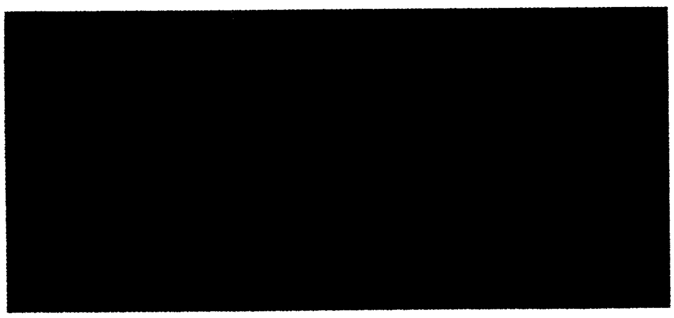
In a postcard to Mr. and Mrs. Max Born in Berlin written several days before he left Japan, Einstein described his impressions on Japan as follows.

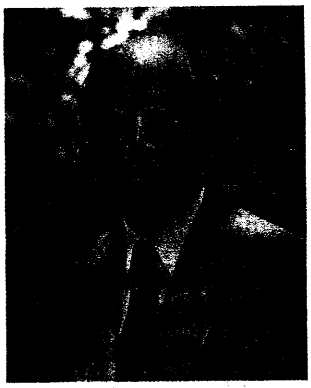
> *"Dear Borns, Splendid sunshine at Christmas. A happy, beautiful country, with a delicate, sensitive people." (23 December 1922.)*

In a letter to Niels Bohr in Copenhagen written aboard ship near Singapore on the way home, he wrote,

> *"The trip is splendid. I am charmed by Japan and the Japanese and sure that you would be, too." (11 January 1923.)*

Prior to visiting Japan, Einstein had toured the Netherlands,





Kaneko Tsutomu

Czechoslovakia, Austria, the United States and Britain in 1920 and 1921, and in the spring of 1922 he had also visited France. These visits, however, had mainly political significance in that he was acting as a cultural envoy or lending support to the Zionist movement, and culturally speaking, though he belonged to the European sphere, where the scars of World War I were still fresh, he was dogged by the Jewish issue. Let me emphasize the fact that at the same time as evoking an extraordinary response towards the Japanese scenery and people, as shown in the above remarks, this trip to Japan, where prejudice against Jews was virtually non-existent, was Einstein's first and last Asian experience, and heightened his awareness as a cosmopolitan.

For an invitation to a single scientist, the Einstein boom in Japan was of an unprecedented scale far exceeding the post-World-War-II boom centering on Yukawa Hideki, Japan's first Nobel Prize winner who was living in America at the time. An account sent to the home office by the German Ambassador in Japan at that time, Wilhelm Solf, illustrated this fact.

> *"His trip to Japan resembled the parade of a general returning from a triumphant campaign. During the visits of the Prince of Wales and General Joffre, the Imperial Court and the military put on a display and there were planned preparations and a semiofficial response on the part of the media. However, there was none of this kind in the welcome accorded to Einstein — on the contrary, the whole Japanese populace from the highest dignitaries down to rickshaw coolies participated spontaneously and without any preparations or compulsion!"*

Cover of the monthly magazine "KAIZO," December issue in 1922, published by Kaizosha Publishing Company. "The Special Issue on Einstein" is mentioned on the cover.

Most Japanese viewed Einstein as the flower of German science, and treated him as a German scientist. In Germany he was attacked as a non-German Jew, but his position in overseas as a representative German scientist was emphasized to exaggeration. Einstein himself was forced to put up with this double-faced Janus-like position about which he could do nothing. At a luncheon hosted by the mayor of Osaka, Einstein's statement that he accepted the elaborate welcome of the German national anthem and German flag not as an individual but in the name of science was probably his own response to this ambivalent situation.

Apparently the manner, in which the Japanese made a cult out of German science, made a great impression on Einstein, as his travel diary (unpublished) reveals.

It was often said that only seven people in the world understood the theory of relativity at that time. Nevertheless, the reason that many people gathered to hear it was because the general intellectual class which supported the wide-spread Taisho democracy movement had expectations of it not as an isolated physics theory, but as an idea opening up new horizons.

During his 43-day sojourn in Japan, an enthusiastic welcome was accorded to Einstein by both the government and the people. Einstein delivered a total of seven general lectures. The admission fee was three yen for adults and two yen for students. This was equivalent to the cost of ten ordinary lunches, so it was a considerable sum. Two of them were in Tokyo and one each in Sendai, Nagoya, Osaka, Kyoto and Fukuoka. Einstein spoke with flavor for five hours on occasion, addressing a full house wherever he went. To give an idea of this wild enthusiasm: several dozens of people who had entrance tickets for the lecture at Kanda in Tokyo were unable to enter because of the great congestion, so Kaizosha Publishing Company paid their return fare to Sendai, the site of the next lecture. Einstein also had some contact, though slight, with ten of the sixteen universities in Japan at that time, attending welcome parties held by students at five universities, and his charm directly left a great impression on Japanese youth.

As mentioned earlier, when Kaizosha's Yamamoto Sanehiko asked Bertrand Russell who are the three greatest people in the world, Russell replied

"First Einstein, then Lenin. There is nobody else."

Welcome at the Tokyo Station Hotel in the Tokyo Station at arrival on November 18, 1922.

Albert Einstein strolling with a twig of flower at Chion-in Temple, Kyoto on December 15, 1922.

These words reveal Russell's deep sympathy with Einstein's selfless and courageous action because Einstein was aiming at solidarity with the human race. This was one factor behind the fever of the Einstein boom. Here the theory of relativity is transformed into a weapon of social thought.

2. THE ROLE OF INTELLECTUALS

2.1. A Hot Argument between Einstein and Yamamoto Senji

Special mention must be made here of the fact that there was a very heated discussion between Einstein and a certain Japanese social thinker over their different evaluation of the intelligentsia. The person involved was Yamamoto Senji, commonly known as Yamasen, who was a biologist and lecturer in the Faculty of Medicine at Kyoto University and popular amongst the people as a social activist. At the same time, he was influenced by Margaret Sanger, and his branching out into political criticism as a pioneer advocate of sex education also occurred around the time of Einstein's visit to Japan in 1922. He evolved a legitimate proletarian movement, and was active as a leftist Diet member representing Kyoto until being stabbed to death by a rightist in 1929.

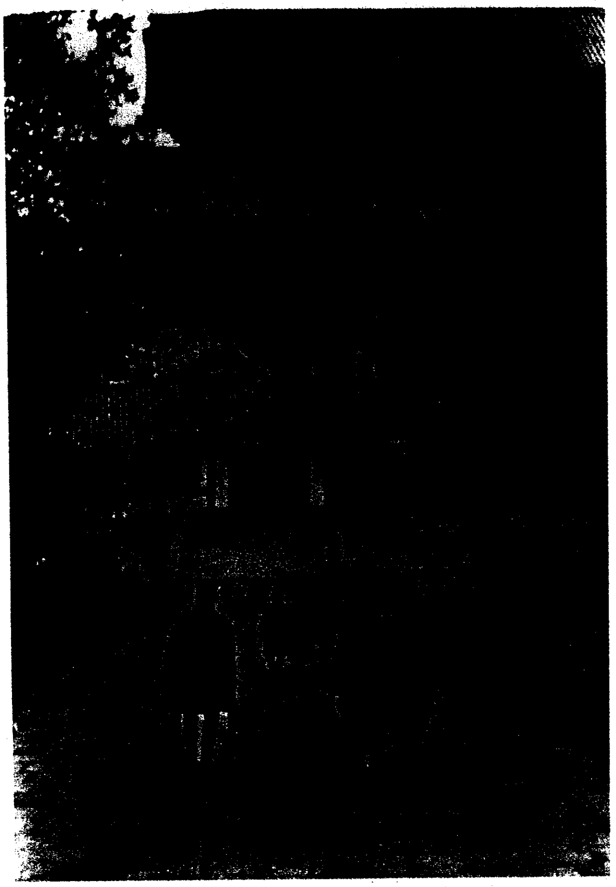
Yamamoto Senji (1889 - 1929)

Yamasen visited Einstein at the Miyako Hotel in Kyoto in order to receive a recommendation for an anti-War book he had translated. After receiving the recommendation,

> *"When we self-interestedly tried to bid farewell, Professor Einstein detained us and requested us to stay for a while, and asked us various questions."*

Thus a discussion developed between Yamasen and Einstein on the role of intellectuals, and this dialogue deserves special mention amongst the events that occurred during Einstein's stay in Japan. Yamasen once described the gist of this discussion.

This dialogue acutely reflects the differences in the viewpoints of Einstein, who held out hope towards the role of the intellectual classes in the peace movement (this is why he accepted the work of the League of Nations Intellectual Cooperation Committee and Yamasen, who rejected this outright as a dream and ceaselessly pinned his hope on the workers' movement rather than the words and actions of scholars who discuss "concepts only that are meaningless in content and uselessly grandiose in outward." Rather than being a mere differ-



Albert and Elsa Einstein (on the right hand side) strolling at Todai-ji Temple, Nara on December 18, 1922.

ence between the political situations of Germany and Japan, this was also a difference between Einstein, who had a global viewpoint, and Yamasen, who was challenging Japanese reality.

Einstein maintained that

> *"The best method for intellectuals to promote international peace and brotherly love amongst mankind is through their scientific contributions and artistic achievements. Creative work raises man high above individual and self-centered national goals."*

Einstein had a consistently high opinion of the role of intellectuals in overcoming nationalism. Meanwhile, Yamasen wrote that

> *"For better or for worse, the Japanese intellectual class is overrated by Professor Einstein."*

and disparaged the Japanese intellectual's emphasis on formality as follows:

> *"The reason is that the necessary conditions belonging to the Japanese intellectual class nowadays are an awareness of oneself as an intellectual and a career and dignity and self-respect appropriate to this position; in other words, self-conceit plus a diploma plus an attitude of stoicism. Naturally, as I said before, this is a question of form alone, and the content is a separate matter."*

A young intellectual who was present at this dialogue between Yamasen and Einstein has written that Einstein threw up his hands and argued furiously in German that

> *"Peace will not come to the world unless you attack the militarism and chauvinism of the Japanese government."*

Unfortunately, Einstein's travel diary makes no mention at all of this memorable meeting.

3. THE PHILOSOPHICAL INFLUENCE

So far our discussion of Einstein's impact on Japan has centered on the socio-cultural aspects. A small space remains for a description of the philosophical and scientific aspects.

There were philosophers who grappled more directly with the issue of the philosophical significance of the theory of relativity. Saegusa Hiroto, who commenced from a study of Emmanuel Kant and Georg W. F. Hegel and evolved a theory of technology, and Tosaka Jun, who together with Saegusa founded a society for studying materialism in 1932 and who became a pioneer of scientific philosophy and died in prison during the World War II, were amongst those who sought the subject matter for their first published papers in the theory of relativity.

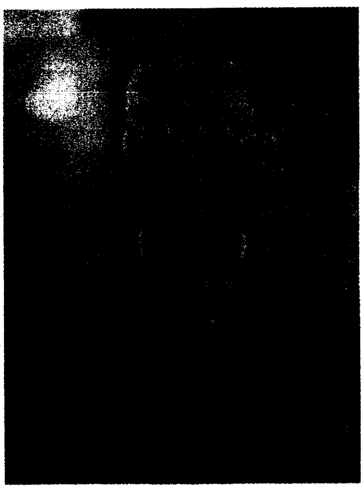
It goes without saying the fact that Nishida Kitaro and Tanabe Hajime were the two greatest authorities in the Japanese philosophical circles at least before the World War II. As described earlier, these two had a major influence on inviting Einstein to Japan, and Tanabe's deep involvement with the theory of relativity in the early formation of his thought continued right through until late in his life. It is evident that at that time Nishida evinced extraordinary interest in the theory of relativity in his correspondence and other writings. This interest bears fruit as the third paper entitled *Hataraku Mono kara Miru Mono e* (from acting to seeing) in his collection of papers. This collection of papers is a compilation of nine papers written over the five years from 1923 to 1927. Since *Zen no Kenkyu* (the study of virtue), Nishida's thought took an enormous leap forward in this work, which is said to have established the so-called "Nishida philosophy" that fuses Oriental Zen thought with Western logic. In this third paper he evaluates the appearance of the theory of relativity as a victory that "skillfully captures the principles that constitute physical knowledge, and penetrates into the truth of physical phenomena." Nishida then claimed that the four-dimensional space-time introduced by this theory is regarded as the objective world of the truly active self, and that this world is divided into two depending on whether it has a temporal content or not. If it is empty time it is regarded as the physical world, and if it is time which has content, it is regarded as the spiritual world. Without doubt, the theory of relativity acted as a turning point for Nishida's philosophy, which was still in the process of formation.

4. THE SCIENTIFIC INFLUENCE

In Japanese scientific circles the theory of relativity had already begun to gain acceptance for quite some time before Einstein's visit to Japan. Einstein's ideas had been widely introduced by the physicist Kuwaki Ayao, who on 17th March 1909 became the first Japanese to meet Einstein, who was working in the patent office in Bern, and by Ishiwara Jun, who in the same year became the first Japanese to publish a paper on the theory of relativity, as well as by Tanabe Hajime and others. This level of enlightenment was quite high, comparing favorably with that in the West.

(3) This was also a period of great change from private science to industry-based science. Before that time scientific research had been carried out in the tiny laboratories of professors or in the back rooms of inventor's homes. However, the period around 1920 saw the arrival of an era of organized science centering on corporate research institutions, which were supported by abundant funds, and on university departments of science and engineering and various national research and test institutions, which were expanded in response to the establishment of the corporate institutions.

Original Japanese scientific research had already appeared in each field in the early twentieth century, and steady progress was continuing towards increased independence. However, this sporadic research was carried out by Japanese studying in Europe, or even if carried out in Japan the research was isolated one in many cases. In order to have a broad and profound influence on each field of science it was necessary to hasten the preparation of an adequate research setup. This was a task facing Japanese scientific and technological circles from



Ishiwara Jun (1881 - 1947)

Albert Einstein signing his name with a ink-filled bold brush at a Japanese Inn in Fukuoka on December 25, 1922.

the 1910s into the 1920s, or throughout the whole Taisho period. Einstein's visit to Japan fitted in with this mood of promoting science and technology. There must have been a great gap in the degree of familiarity with science on the part of the general public in America, where Thomas Edison, who had become a national hero through the 19th century science and his inventions, was still alive, and Japan, which lacked such a personage. A certain Tohoku University assistant professor who attended one of Einstein's university lectures wrote to a local newspaper that he felt such fondness towards Einstein that he felt like calling him "Father". This term also symbolizes the relationship between the Japanese scientific world and that of Europe, at the top of which stood Einstein. This topological relation is even carried through in the welcoming address given at the Imperial Academy.

This was also apparent in the series of lectures given at the University of Tokyo. Extending over a total of five days, Einstein gave specialized lectures on the theory of relativity, but their content was virtually identical to the lectures he had given at Princeton University more than a year ago, a record of which had already been published. Truly regrettable for the Japanese physics world is the fact that instead of drawing out in this lecture the full faculties of their distinguished guest, Japanese scientists meekly let slip this once-in-a-lifetime opportunity by taking the attitude of a master-apprentice relationship, despite the fact that some discord was caused by the appearance of Doi Fuzumi, who opposed the theory of relativity. In this respect the lecture at Kyoto University was a success in that, at the request of Nishida Kitaro, it was a per-

Albert and Elsa Einstein departing from Japan on N. Y. K. liner, Haruna-maru, at Moji Port on December 29, 1922.

sonal and vivid description of how Einstein developed his theory of relativity.

In a letter to a friend, Terada Torahiko, a physicist well-known for his essays, who was awarded later the Japan Academy Prize, lamented the fact that the older generation were monopolizing Einstein. The fact that no opportunity was provided for Terada and Ishiwara and the younger generation to hold a free discussion with Einstein was also related to the traditional Japanese attitude towards learning. The Confucian precedence of the elder over the younger was still strict, and learning was regarded mainly as knowledge leading to success, and the general intellectual climate lacked consideration for learning as a process.

In closing, let me emphasize that Einstein's visit to Japan had a major influence on the young generation including Yukawa Hideki and Tomonaga Sin-itiro, who were still at junior high school, and acted as a stimulus producing many physicists.