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“A Brief Summary of My Research”

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I have mainly worked four fields of theoretical physics of elementary particles.

- Topological quantum field theories
- Quantum gravity
- Brane world
- Superstring theories

1. Topological quantum field theories

- Witten's topological field theories for 4D Donaldson invariants (Witten's type or cohomological type)

Classical action: $$S_c = 0$$

- Schwarz's topological field theories for Jones' knot invariants or Ray-Singer torsions (Schwarz's type or Chern-Simons type)

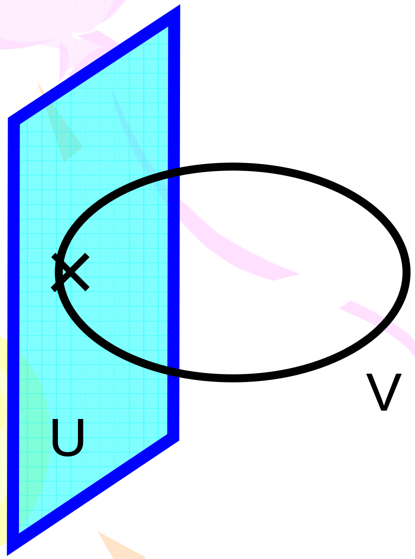
Classical action: $$S_c = Tr(AdA + \frac{2}{3}A^3)$$

Characteristic feature → Non-existence of the metric tensor in classical action

Topological BF theory (Schwarz's type)

Classical action: $$S_c = \int_{M_D} Tr B_{D-2}(dA + A^2)$$

1. Linking numbers S. Yahikozawa and I. O., P.L.B238(1990)272



$$< \int_U B \int_V A >$$

$$= \int DB \int DA \int_U B \int_V A \; e^{iS}$$

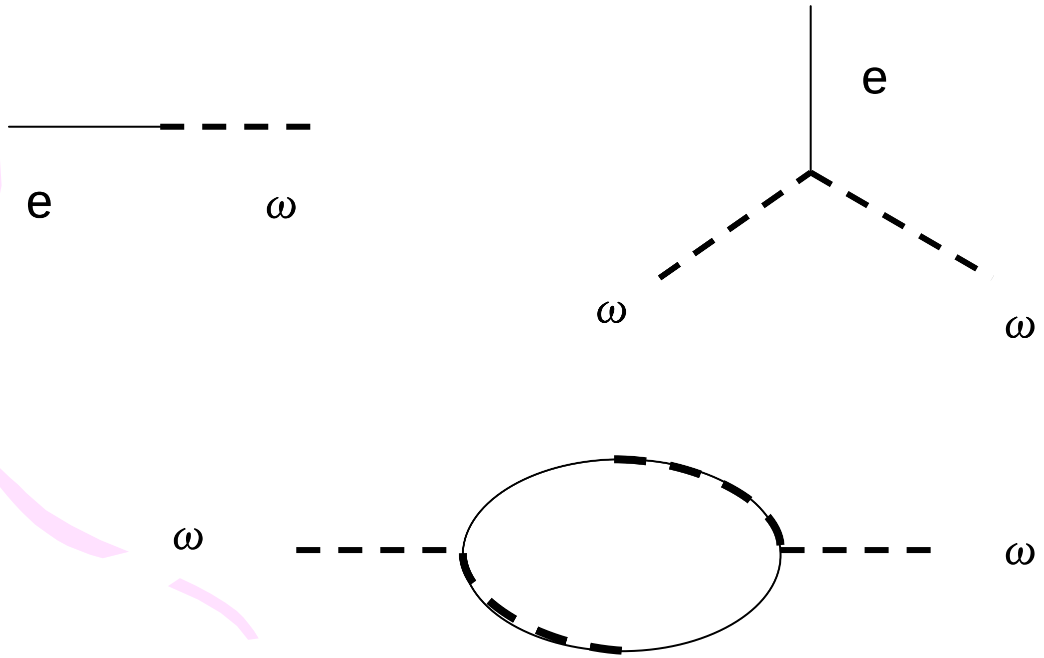
$$= iL(U,V)$$

L(U,V): linking number.

2. 3D Einstein gravity Gauge group=ISO(1,2)

S. Yahikozawa and I. O., C.Q.G 11(1994)2653

$$S_c = \int_{M_3} Tr\ e(d\omega + \omega^2)$$



$$S_q = k \int_{M_3} Tr\ (\omega d\omega + \frac{2}{3}\omega^3)$$

3. Topological Higgs mechanism

S. Yahikozawa and I.O., P.L.B234 (1990) 69

What becomes if we cannot find Higgs particles in future?!

An alternative mechanism → Topological Higgs mechanism

- 3D Chern-Simons

$$S = -\frac{1}{4}\int F^2 + m\int A\wedge dA$$

$$\Rightarrow \quad (\partial^2 - m^2)A_\mu = 0$$

- BF theory in arbitrary space-time dimensions

$$S = -\frac{1}{4}\int F^2 - \frac{1}{4}\int H^2 + m\int A\wedge dB$$

$$\Rightarrow \quad (\partial^2 - m^2)A = (\partial^2 - m^2)B = 0$$

Many U(1) gauge fields with BF theory-like coupling appear in string theory!

4. BRST quantization

M. Tonin and I. O., P.L. B623 (2005) 155

On-shell reducible constraints ⟶ Batalin-Vilkovisky formalism

5. Topological 4D Einstein gravity

A. Sugamoto and I. O., P.L. B266 (1991) 280

Topological Yang-Mills theory ⟶ Donaldson invariants

Topological Einstein gravity ⟶ Gravitational version of Donaldson invariants?

6. Topological pregeometry

K. Akama and I. O., P.L. B259 (1991) 431

Generation of dynamics from topological theory

2. Quantum gravity — Quantum black holes—

- 2D dilaton gravity

Classical action:

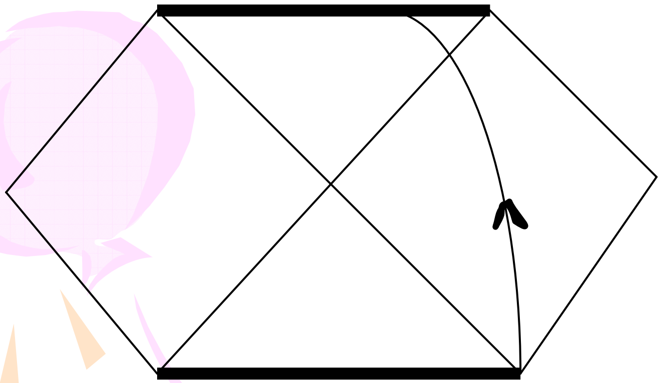
$$S_c = \frac{1}{2\pi}\sqrt{-g}[e^{-\phi}(R + 4(\nabla\phi)^2 + 4\lambda^2)]$$

1. A new solution with E. M. field
2. A new formalism based on SL(2,R)/U(1) coset model
3. Disappearance of quantum black holes with supersymmetry
4. Wormhole solutions

S. Nojiri and I. O., P.L.B294 (1992) 317; N.P.B406 (1993) 499; M.P.L. A8 (1993) 53; P.R.D48 (1994) 4066; M.P.L. A9 (1994) 959

- 4D Einstein gravity

A. Hosoya and I. O., P.T.P. 97 (1997) 233



What is a spacetime singularity?

Geodesic incompleteness

⟶ Penrose-Hawking singularity theorem

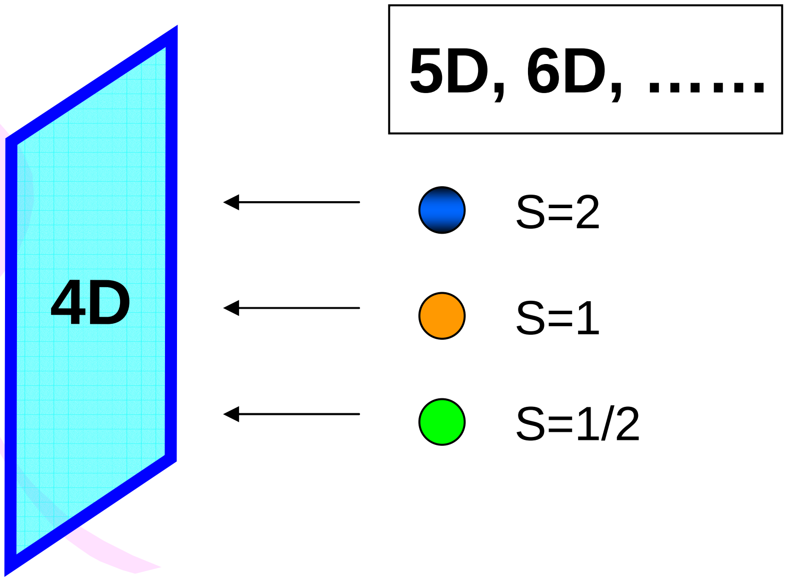
“Quantum” generalized affine parameter:

$$|\alpha|_q = \int_0^1 dt \sqrt{\sum_{a,b=0}^{3} < [V^\mu e_\mu^b (P exp \int_0^t \omega)_b^a]^2 >}$$

$|\alpha|_q \rightarrow \infty \Longrightarrow$ Wiping up of singularity in quantum gravity

3. Brane world

An alternative resolution to hierarchy problem
and Kaluza-Klein compactification



Problem: Difficult to localize **gauge fields with spin 1** in 5D brane world!

⇒ **6D brane world**

I. O., P.L. B496 (2000) 113; P.R.D62 (2000) 126009

4. Superstring theories

Only consistent quantum gravity and a strong candidate of Theory of Everything (TOE)

A big remaining problem is to understand the non-perturbative, strong-coupling phase!

Plausible approaches:

1. String field theories ⟶ only bosonic open string
2. Various string dualities ⟶ only region near weak-coupling phase
3. D-brane ⟷ stringy instanton carrying non-perturbative information

Problem: Difficult to calculate amplitudes with D-branes within the conventional Neveu-Schwarz-Ramond (NSR) formalism

A longstanding problem since 1984:
"Covariant quantization of Green-Schwarz superstring action"

Neveu-Schwarz-Ramond(NSR) superstring theory

→
- Lorentz-covariant
- 2D world-sheet supersymmetry manifest
- 10D space-time supersymmetry only after GSO projection
- Difficult to deal with R-R fields which are the carriers of D-brane

Green-Schwarz(GS) superstring theory

→
- Lorentz-covariant at least classically
- 10D space-time supersymmetry manifest
- Possible to deal with R-Rfields on the equal footing with NS-NS fields

⇩

Difficult to perform the covariant quantization owing to κ symmetry

⇩

Non-covariant **light-cone gauge**

A new approach by Berkovits in 2000:
"Pure spinor quantization of Green-Schwarz superstring action"

What is the pure spinor? E. Cartan, 1930's

$$\lambda^a(\Gamma^{\mu_1\cdots\mu_n})_{ab}\lambda^b = 0$$

Here λ^a is complex, commuting spinor in even dimensions.

In 10D,

$$\lambda^a(\Gamma^\mu)_{ab}\lambda^b = 0$$

BRST charge:

$$Q = \oint \lambda^a d_a$$

$$d_a(y)d_b(z) \to -\frac{1}{y-z}(\Gamma^\mu)_{ab}\Pi_\mu$$

$\longrightarrow$ $$Q^2 = -\oint \lambda^a(\Gamma^\mu)_{ab}\lambda^b\Pi_\mu = 0$$

$\longrightarrow$ BRST cohomology

Action:

$$S = \int d^2z\left(\frac{1}{2}\partial X^{\mu}\bar{\partial}X_{\mu} + p_a\bar{\partial}\theta^a + \omega_a\bar{\partial}\lambda^a\right)$$

But there are many cruxes in this approach in 2000.

1. Q contains both 1-st and 2-nd class constraints, whereas the conventional Q involves only 1-st class one.

2. Q is nilpotent only when the pure spinor condition is used.

3. What is the relation between the pure spinor action and the GS action?

M. Tonin and I. O., P.L. B520 (2001) 398

4. What is the origin of the pure spinor formalism?

e.t.c.

M. Matone, L. Mazzucato, D. Sorokin,
M. Tonin and I. O., N.P. B639 (2002) 182

Pure spinor quantization of Green-Schwarz superstring

Present status:

All nontrivial tests are passed!
The famous problem for about 20 years has been now solved.

Future prospect:

- Calculations of higher-loop amplitudes
- Superstring field theory based on pure spinor formalism
- Understanding the non-perturbative string physics
- Application to 4D string theory

e.t.c.

M. Tonin and I. O., P.L. B606 (2005) 218; P.L. B623 (2005) 155; N.P. B727 (2005) 176-195