Computational physics mini-course

Problems for Part II, Physics 903, Fall/2006.

Solve 2 problems below. Due date 12/18/2006.

N–1. Consider a linear PDE, $\partial_t f + c\partial_x f = 0$ with c>0.
a) Derive the Warming & Beam scheme and check the order of truncation error.
b) Apply von Neumann analysis to the Warming & Beam scheme.

N–2. Consider the same linear PDE as Problem 1.
In the form of finite volume discretization, $w_j^{n+1} = w_j^n - \frac{\tau}{h}\left(F_{j+\frac{1}{2}}^n - F_{j-\frac{1}{2}}^n\right)$.
a scheme
with a flux $F_{j+1/2}^n = \frac{1}{2}(F_{j+1/2}^n[\text{Lax-Wendroff}] + F_{j+1/2}^n[\text{Warming \& Beam}])$
is called Fromm scheme.
a) Derive the Fromm scheme and check the order of truncation error.
b) Apply von Neumann analysis to the Fromm scheme.

N–3. Write a numerical code to solve a linear PDE $\partial_t f + c\partial_x f = 0$, using one or two numerical scheme introduced in the lecture, or any other schemes you are interested. Perform simulations starting from a initial data with a discontinuity such as $\begin{cases} f(0,x) = 1, & \text{for } x \le 0 \\ f(0,x) = 0, & \text{for } x > 0 \end{cases}$, then discuss the property of the scheme.

N–4. Prove the monotonicity preserving theorem and the order barrier theorem for the linear multi-step s-step scheme, $w_j^{n+1} = \sum_{p=0}^{s-1}\sum_m c_{m,p}\, w_{j+m}^{n-p}$

G–1. Explain the roles of the lapse function, shift vector, and extrinsic curvature in 3+1 decomposition of globally hyperbolic spacetime.

G–2. Prove the Gauss-Codazzi equation,

$$ {}^3R_{\alpha\beta\gamma\delta} = h^{\mu}{}_{\alpha}h^{\nu}{}_{\beta}h^{\sigma}{}_{\gamma}h^{\rho}{}_{\delta}R_{\mu\nu\sigma\rho} + K_{\alpha\delta}K_{\beta\gamma} - K_{\alpha\gamma}K_{\beta\delta} $$

G–3. Prove a relation

$$ R_{\varepsilon\iota\kappa\lambda}h^{\varepsilon}{}_{\alpha}h^{\kappa}{}_{\beta}n^{\iota}n^{\lambda} = -\pounds_n K_{\alpha\beta} + K_{\alpha\gamma}K_{\beta}{}^{\gamma} + \frac{1}{\alpha}D_{\alpha}D_{\beta}\alpha $$

then derive a spatial projection of Einstein tensor

$$ \begin{aligned} G_{\gamma\delta}h^{\gamma}{}_{\alpha}h^{\delta}{}_{\beta} \;=\; & \pounds_n K_{\alpha\beta} - h_{\alpha\beta}\pounds_n K + {}^3R_{\alpha\beta} - \frac{1}{2}h_{\alpha\beta}{}^3R \\ & + KK_{\alpha\beta} - 2K_{\alpha\gamma}K_{\beta}{}^{\gamma} - \frac{1}{2}h_{\alpha\beta}(K^2 + K_{\gamma\delta}K^{\gamma\delta}) \\ & - \frac{1}{\alpha}D_{\alpha}D_{\beta}\alpha + \frac{1}{\alpha}h_{\alpha\beta}D_{\gamma}D^{\gamma}\alpha \end{aligned} $$

Note that ${}^3R_{\alpha\beta\gamma\delta}$, ${}^3R_{\alpha\beta}$, ${}^3R$, are, respectively, the curvature tensor, Ricci tensor, and scalar curvature associated with spatial metric $h_{\alpha\beta}$, and ${}^3R_{\alpha\beta\gamma\delta}$, is the curvature tensor associated with the spacetime metric $g_{\alpha\beta}$.