Short introduction for the quasi-equilibrium binary neutron star solutions.

- Introducing two patches of fluid coordinate grids, the initial data code can be extended for the binary neutron star code.

- Such initial data is also called as the (quasi-)equilibrium circular solution of the binary neutron stars.

- A sequence of equilibrium solutions with constant baryon mass, constant entropy, and constant circulation for zero viscosity case (or a corotational flow for the strong viscosity case) models an inspiral of binary neutron stars.

- A condition for the constant entropy implies the one-parameter EOS, and that for the constant circulation a restriction to the flow field that is included in the hydrostationary equation by construction. Then we left with computations for a set of solutions that has the same rest mass for each NS.

- For the inspiraling binary neutron stars, the flow field is expected to become irrotational. If the neutron star matter has a very strong viscosity, the flow field may become corotational, but this is considered to be unlikely.

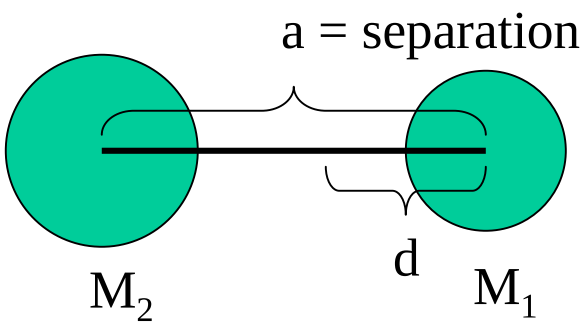
Constant rest mass sequence for unequal mass binary neutron stars.

- We write the rest mass and the central density of each NS component as (M_1, ρ_1), and (M_2, ρ_2).

- To compute a solution that models the inspiral, we need to adjust three parameters, the rest mass M_1 and the ratio of the rest mass $q = M_2/M_1$ to be desirable values, and y-component of the linear momentum P_y to be zero, by adjusting the central densities ρ_1 and ρ_2, and the center of circular orbit.

$$P_y := -\frac{1}{8\pi}\int_\infty \pi^a{}_b \hat{y}^b \, dS_a = \frac{1}{8\pi}\int_\infty K^a{}_y \, dS_a$$

$$P_y = \frac{1}{8\pi}\int_\Sigma 8\pi j_y \psi^6 \sqrt{f} d^3x$$



- For the adjustment of these parameters to desirable values, the discrete Newton-Raphson method appears to be useful.

Recall: Newton-Raphson method; Find a solution for $F(x) = 0$, where F and x may be the vector valued, and each component of $F(x)$ may be nonlinear.

Suppose $x^{(n)}$ is an approximation of a true solution, and $x^{(n)} + \delta x$ is exact, $F(x^{(n)} + \delta x) = 0$. Expanding this to the first order, we have

$$F(x^{(n)} + \delta x) \approx F(x^{(n)}) + \partial F(x^{(n)})/\partial x \, \delta x \approx 0.$$

Therefore we perform the following iteration;

$$\delta x^{(n)} \begin{cases} \delta x^{(n)} := [\partial F(x^{(n)})/\partial x]^{-1} \bullet [-F(x^{(n)})] & \longleftarrow \text{If the form of the inverse of the Jacobian is known.} \\ \text{Or, } \partial F(x^{(n)})/\partial x \bullet \delta x^{(n)} = -F(x^{(n)}) & \longleftarrow \text{Or solve this linear eq.} \end{cases}$$

Update: $x^{(n+1)} = x^{(n)} + \delta x^{(n)}$

The method is called Newton-Raphson if the Jacobian $\partial F(x^{(n)})/\partial x$ is computed analytically, while it is called secant method if the Jacobian $\partial F(x^{(n)})/\partial x$ is computed by the finite difference formula.

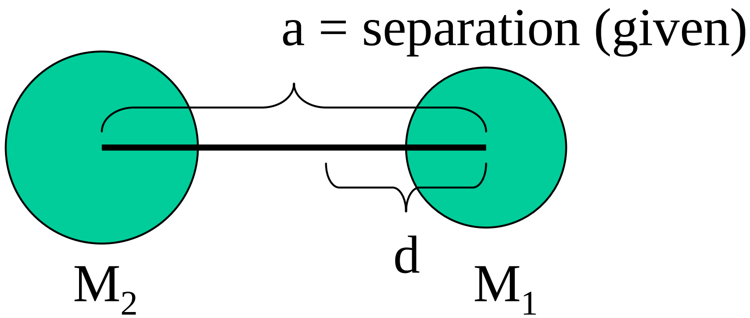
If the form of the function F is not given explicitly at all, we use the Discrete Newton-Raphson method described in the next page.

Discrete Newton-Raphson method

$F_i(x_k) = 0$

$F_i = (M_1, q, P_y)$

$x_k = (\rho_1, \rho_2, d)$



For finding desirable values for (M_1, q, P_y), we adjust (ρ_1, ρ_2, d). However, explicit forms of (M_1, q, P_y) in terms of (ρ_1, ρ_2, d) are not given.

Discrete Newton-Raphson method uses the Jacobian calculated in the following manner:

For n functions F_i , i = 1, …, n and n parameters x_k , k = 1, …, n, compute $F_i(x_k^{(n)})$, and $F_i(x_k^{(n)} + \varepsilon_j \delta_k^j)$ for j = 1, …, n, where δ_k^j is a Kronecker delta, and ε is a small value ($\varepsilon_j \sim 10^{-8} x_j^{(n)}$ is recommended in a book. I'm using $10^{-4} x_j^{(n)}$). Then, the Jacobian is calculated from

$$\frac{\partial F_i(x)}{\partial x_j^{(n)}} = \frac{F_i(x_k^{(n)} + \varepsilon_j \delta_k^j) - F_i(x_k^{(n)})}{\varepsilon_j}$$

Therefore one needs n+1 converged solution to compute a Jacobian.